Chapter 3
STRAIN

By:
Ardiyansyah Syahrom

Department of Applied Mechanics and Design
Faculty of Mechanical Engineering
Universiti Teknologi Malaysia

CHAPTER OBJECTIVES

- Define concept of normal strain
- Define concept of shear strain
- Determine normal and shear strain in engineering applications

CHAPTER OUTLINE

1. Deformation
2. Strain

2.1 DEFORMATION

Deformation

- Occurs when a force is applied to a body
- Can be highly visible or practically unnoticeable
- Can also occur when temperature of a body is changed
- Is not uniform throughout a body's volume, thus change in geometry of any line segment within body may vary along its length

2.1 DEFORMATION

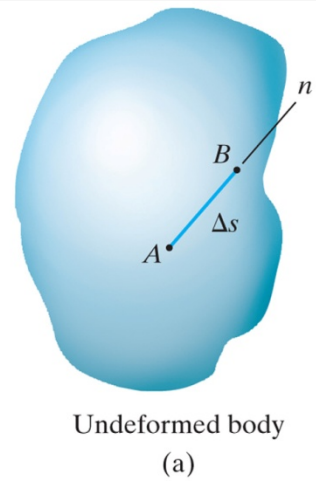
To simplify study of deformation

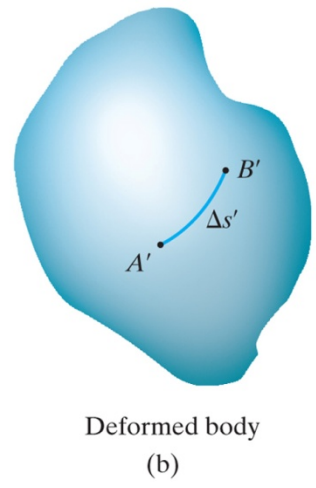
- Assume lines to be very short and located in neighborhood of a point, and
- Take into account the orientation of the line segment at the point

2.2 STRAIN

Normal strain

- Defined as the elongation or contraction of a line segment per unit of length
- Consider line *AB* in figure below
- After deformation, Δs changes to $\Delta s'$





2.2 STRAIN

Normal strain

- Defining *average normal strain* using ε_{avg} (epsilon)

$$\varepsilon_{avg} = \frac{\text{final length - original length}}{\text{original length}} = \frac{\Delta s' - \Delta s}{\Delta s}$$

2.2 STRAIN

Units

- Normal strain is a *dimensionless quantity*, as it's a ratio of two lengths
- But common practice to state it in terms of meters/meter (m/m)
- ε is small for most engineering applications, so is normally expressed as micrometers per meter (μm/m) where $1\ \mu\text{m} = 10^{-6}$
- Also expressed as a percentage, e.g., $0.001\ \text{m/m} = 0.1\ \%$

EXAMPLE 2–2

A force acting on the grip of the lever arm shown in Fig. 2–5*a* causes the arm to rotate clockwise through an angle of $\theta = 0.002$ rad. Determine the average normal strain developed in the wire *BC*.

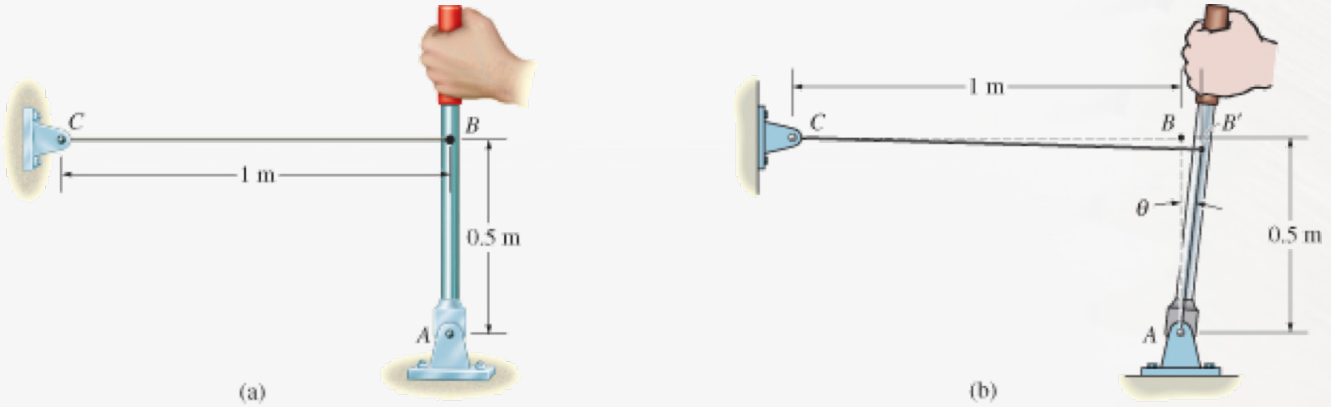


Fig. 2–5

SOLUTION

Since $\theta = 0.002$ rad is small, the stretch in the wire CB, Fig. 2–5*b*, is $BB' = \theta\,(0.5\text{ m}) = (0.002\text{ rad})(0.5\text{ m}) = 0.001\text{ m}$. The average normal strain in the wire is therefore,

$$\epsilon_{\text{avg}} = \frac{BB'}{CB} = \frac{0.001}{1\text{ m}} = 0.001\text{ m/m} \qquad \textit{Ans.}$$

2.2 STRAIN

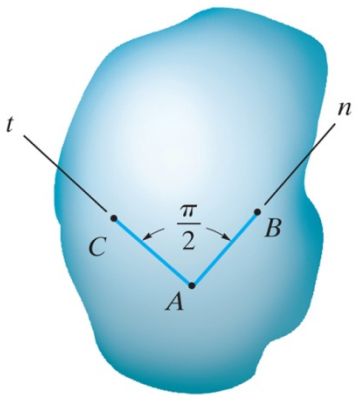
Shear strain

- Defined as the *change in angle* that occurs between two line segments that were originally *perpendicular* to one another
- This angle is denoted by γ (gamma) and measured in radians (rad).

2.2 STRAIN

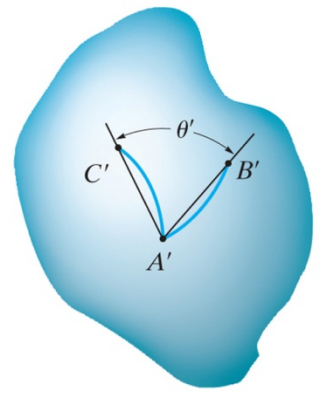
Shear strain

- Consider line segments AB and AC originating from same point A in a body, and directed along the perpendicular n and t axes
- After deformation, lines become curves, such that angle between them at A is θ'



Undeformed body

(a)



Deformed body

(b)

2.2 STRAIN

Shear strain

- Hence, shear strain at point *A* associated with *n* and *t* axes is

$$\gamma_{nt} = \frac{\pi}{2} - \lim_{\substack{B \to A \text{ along } n \\ C \to A \text{ along } t}} \theta'$$

- If θ' is smaller than $\pi/2$, shear strain is positive, otherwise, shear strain is negative

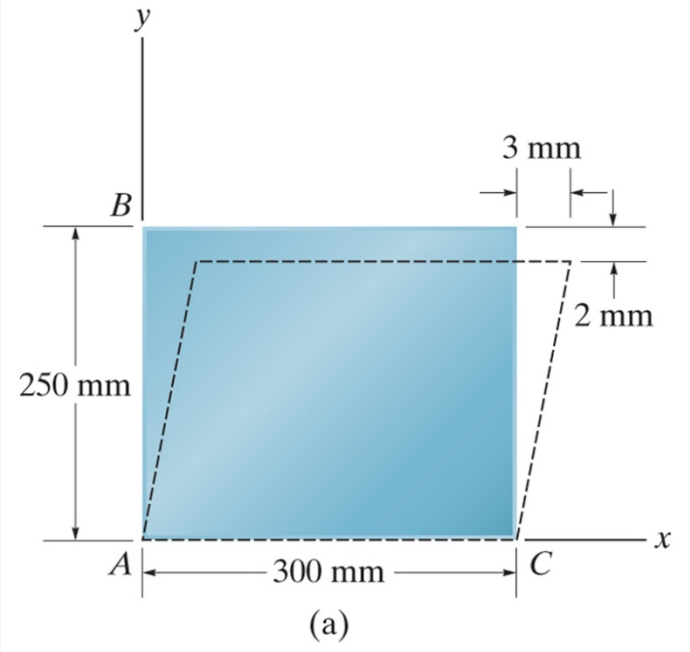
EXAMPLE 2.3

Plate is deformed as shown in figure. In this deformed shape, horizontal lines on the plate remain horizontal and do not change their length.

Determine

(a) average normal strain along side AB,

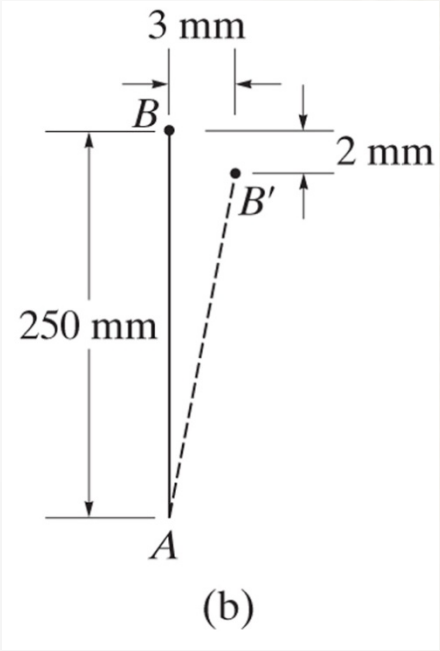
(b) average shear strain in the plate relative to x and y axes



EXAMPLE 2.3 (SOLN)

(a) Line *AB*, coincident with *y* axis, becomes line *AB'* after deformation. Length of line *AB'* is

$$AB' = \sqrt{(250-2)^2 + (3)^2} = 248.018\,\text{mm}$$



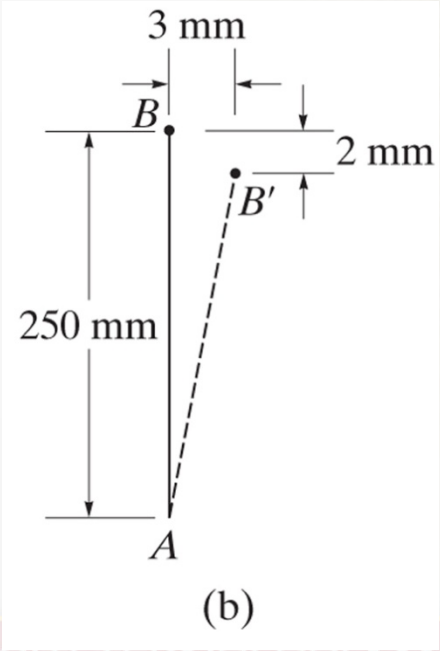
EXAMPLE 2.3 (SOLN)

(a) Therefore, average normal strain for *AB* is,

$$(\varepsilon_{AB})_{avg} = \frac{AB' - AB}{AB} = \frac{248.018\ \text{mm} - 250\ \text{mm}}{250\ \text{mm}}$$

$$= -7.93(10^{-3})\ \text{mm/mm}$$

Negative sign means strain causes a contraction of AB.



EXAMPLE 2.3 (SOLN)

(b) Due to displacement of B to B', angle BAC referenced from x, y axes changes to θ'. Since $\gamma_{xy} = \pi/2 - \theta'$, thus

$$\gamma_{xy} = \tan^{-1}\left(\frac{3\text{ mm}}{250\text{ mm} - 2\text{ mm}}\right) = 0.0121\text{ rad}$$