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A METHODOLOGY FOR RANKING OF ALARMS IN CONTROL CHARTS

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ABSTRACT

Control charts are main tools used to detect whether variation in the quality characteristic is because of chance or assignable cause. Traditional control chart is separated into definite zones and engineers use them as a criterion to either continue or stop the process depending upon the position of the newly observed points in the zones. Since the control charts are defined using zones, points that are in the same zone are viewed as having the same risk of being out of control. This paper discusses the limitations of the traditional zoning of control charts and problems faced by engineers in taking corrective actions based on the results of the charts. A new methodology is proposed in this paper for estimating the risk of the points lying on a control chart. A severity number is then assigned to the points to show the degree of severity of each point. The proposed severity number will be useful for engineers in generating different action plans. Engineers will actually place different approach or rigorousness in investigating for any assignable causes based on the different ranking of the severity number. Later, a simulation is used to show how the proposed method is used in ranking the out of control points.

Keywords: *Control chart, statistical process control (SPC), graded SPC, and decision making using zones.*

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1.0 INTRODUCTION

Statistical Process Control (SPC) is a very popular and useful methodology used by many different industries. In the SPC methodology, control chart is the tool that is widely used for process monitoring. Within the control chart, statistical summary are computed from an identified sample systematically, and are then plotted on a suitable control chart for the purpose of monitoring the ongoing process. Control chart uses pre-calculated upper and lower control limits which are used as a criterion for detecting any abnormality in a process. A phenomenon is said to be under control when, through the use of past experience, it can be predicted, how the phenomenon is expected to vary in the future at least within the set limits. Prediction within limits means the probability that the observed phenomenon will fall within the given limits [1].

Shewhart [1] has mentioned that constant systems of chance causes do exist in nature. This phenomenon is defined as chance. He also recognized the presence of unknown causes of variability that do not belong to a constant system. These causes are called assignable causes.

1.1 Structure of traditional control charts

Control charts are basically run charts (data points plotted against time) but contain added features, the Upper Control Limit (UCL), the Central Line (CL), and the Lower Control Limit (LCL). The structure of a control chart in terms of UCL, CL, and LCL is shown in Figure 1. The estimation of UCL and LCL are from a set of data intended to estimate within a limit the random behavior of a process (chance causes). Each time points lie beyond the limit of the UCL and LCL, the control chart detects a potential assignable cause of variation in the quality characteristic.

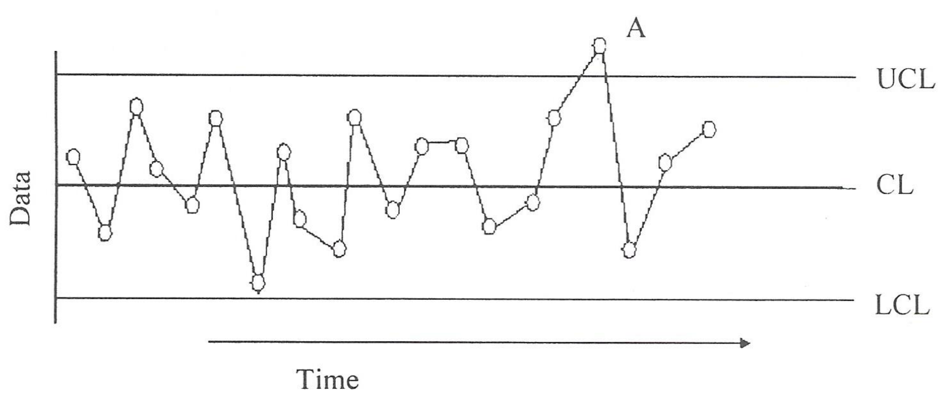

Figure 1: Structure of a control chart

1.2 Limitation of the Traditional Control Chart

The traditional control charts combined with the Western Electric Company (WECO) rules applies zones to define criteria in detecting any abnormality in any process. Each time points appear beyond the UCL , LCL or fails any Western Electric Company (WECO) rules it is considered as an alarm. Nevertheless, there are times that no assignable causes are found and the process is then continued and the next subsequent points would then be the within the UCL and LCL. This phenomenon is commonly known as false alarm or Type 1 error [2]. From the literature study it is found that numerous works has been done in terms of the number of alarms and false alarms. Works from Margavio *et al.* [3], discusses on the control plan using desired pattern of in-control false alarm rates in conjunction with a desired average run length. Bai and Lee [4] discusses about the economic view point of the cost of false alarms, the cost of detecting and eliminating an assignable cause, the cost associated in production in out of control state and the cost of sampling and testing. Reynolds and Lu [5] discusses about the effects of high false alarms on Auto-correlated processes. Chan *et al*, [6] discusses on a methodology to economically calculate false alarms for low fraction non-conforming (p-charts and np-charts). However when there is any alarms or many alarms there are no work done to the best of the authors knowledge that graded the severity of the alarms relative to each other. The traditional control charts combined with the Western Electric Company (WECO) rules uses zones to define criteria to detect any abnormality in any process.

Figure 2 shows a common behaviour of a control chart where two points i.e. A and B are shown out of control limits (UCL and LCL). Anyone with control chart background could identify that at points A and B there is a possibility of the presence of assignable causes. An engineer, in practice will acknowledge these two points as out of control. On the other hand, these two points possess different probability of finding out any assignable cause. The engineer will consider point B to have higher probability of finding any assignable cause compared to point A and will actually give more attention and priority to point B. The limitation of the traditional control chart methodology is that there is no indicator or rank in terms of degree of severity of any points out of control. Points A and B shown in Figure 2 are considered to have the same probability of being out of control.

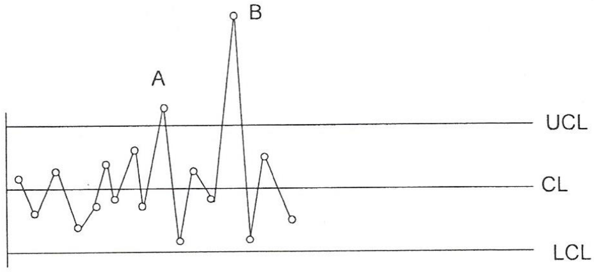

Figure 2: Behavior of a control chart 1

Figure 3 shows another common behaviour of a control chart involving two groups of points Group 1 and Group 2. For this case it is a violation of test 5 (two of three points in zone A or beyond) in the WECO run rules. The traditional control chart will flag alarms for this two groups. However it does not indicate the degree of severity for both of these groups relative to each other.

Based on the engineering experience of the authors it is clear that the first group of points (Group 1: Refer to Figure 3) displays a much less possibility of the presence of assignable causes. In contrast, the second group of points (Group 2: Refer to Figure 3) has a higher possibility of the presence of assignable causes. An engineer will adopt different approach or rigorousness in investigating any assignable causes for these two Groups. The engineer may treat Group 1 as having low possibility of detecting any assignable causes. The process may be stopped and visual inspection would be performed. The engineer will then continue the process and monitor the control chart closely. On the other hand, for Group 2, the engineer will actually stop the process and perform a more thorough inspection. The engineer may dismantle and inspect some of the machine parts and perform more thorough inspection and test.

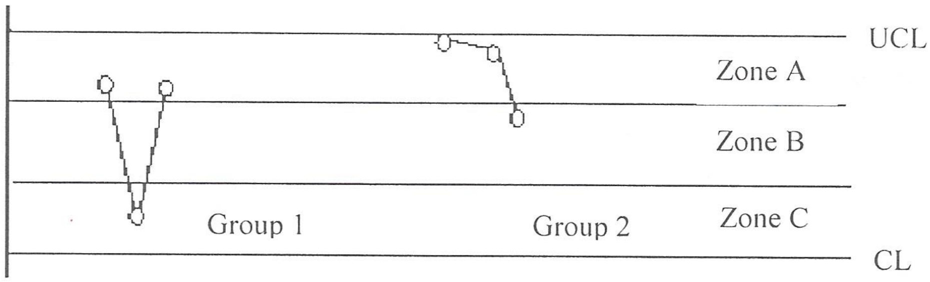

Figure 3: Behavior of a control chart 2

From these two scenarios (Figures 2 and 3), the limitation of the traditional control chart is that it does not have indicators for the degree of severity of alarms. This paper proposes the Graded Severity Ranking (GSR) to address the limitations of the traditional control chart discussed earlier.

2.0 GRADED SEVERITY RANKING ON STATISTICAL PROCESS CONTROL (SPC) CHARTS

The aim of this paper is to propose a methodology of assigning a severity ranking number or indicator to all of the points or combination of points to illustrate the degree of severity of the points that are present in the control chart. Figure 4 shows that points A and B are out of control. Arbitrary GSR 3.1 and 5.2 have been assigned to the points respectively. Later, the actual methodology of estimating GSR of a point in the control chart is proposed. A sample calculation is also discussed.

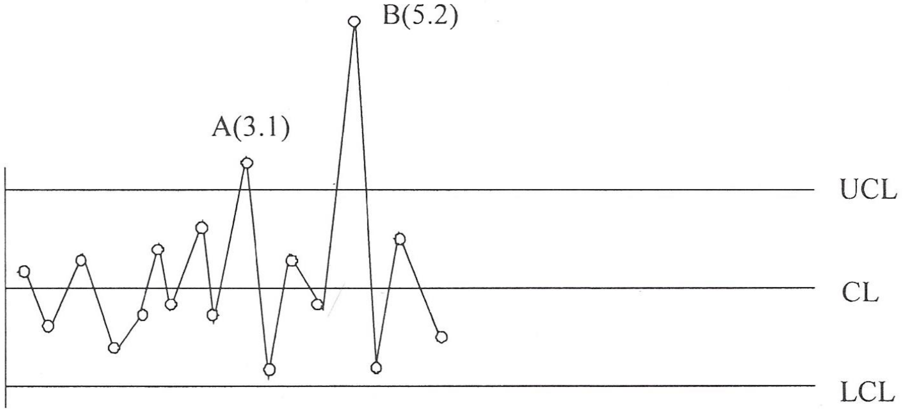

Figure 4: Points A and B are out-of-control

Now the engineer has additional information on the magnitude of out of control of these two alarms relative to one another. This information could help the engineer to take more appropriate action on the process. He will use different approach or rigorousness in generating action plans or finding assignable cause of the process.

Consider the second example in Figure 5. Here, two groups of points Group 1 and Group 2 having arbitrarily assigned GSR are 3.34 and 4.89 respectively. It is clear that points from Group 1 have lower possibility of discovering any assignable cause compared to those of Group 2.

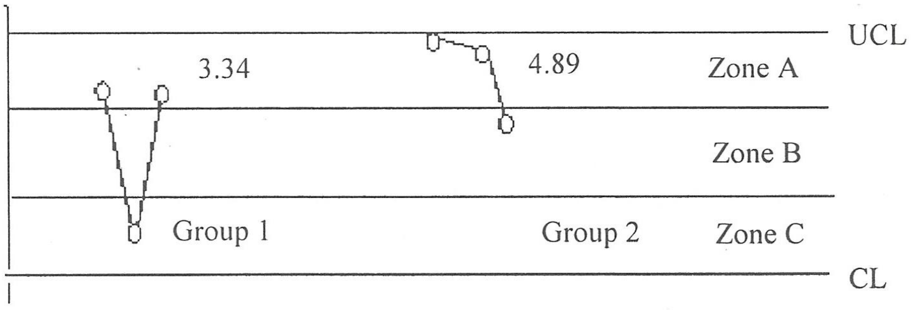

Figure 5: Group 1 and Group 2 are out-of-control

2.1 Proposed Methodology of Estimating the Graded Severity Ranking (GSR)

In this section the methodology of estimating the GSR of the points in a control chart is proposed. The estimation methodology is derived on the foundations of the normal distribution. In a nut shell, for each of the points that appear in the control chart, a probability of that point not being from the normal distribution is estimated. Then the probability of the points appearing in the particular sequence is also taken into consideration. Since the estimated probability is in decimal points (e.g. 0.9998877), the estimated probability is then transformed into a number called GSR. This is done because for an engineer it is, indeed difficult to visualize and interprete numbers like 0.9998877 whereas, it is easier for them to visualize and interpret the GSR.

A four steps approach is used to estimate the probability. They are One Point Estimate (1PE), Two Points Estimate (2PE), Three Points Estimate (3PE) and Four Points Estimate (4PE). Consider Figure 6, the fresh point is point 1. For 1PE, probability of point 1 not being from normal distribution is estimated. Then point 1 is combined with point 2, and the probability of this sequence not being from normal distribution is then estimated. This is defined as 2PE. Next, point 1, point 2 and point 3 are combined and the probability of this sequence not being from normal distribution is estimated. This is defined as 3PE. Lastly, point 1, point 2, point 3 and point 4 are then combined together and the probability of this sequence not being from normal distribution is estimated. This is defined as 4PE. The following section describes in detail these point estimates.

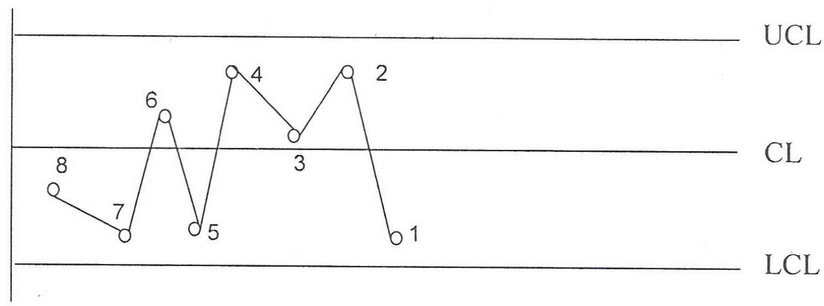

Figure 6: Sequences of Points

2.1.1 One Point Estimate (1PE)

Basic assumptions for a stable process (only has chance causes) are all points follow the approximation of a normal distribution. Suppose a point x_1 collected from a stable process at t_1 a probability value could be estimated based on the normal distribution. It is possible to calculate the probability of value more than x_1. Referring to Figure 7, the probability of point more than x_1 is the area shaded, where μ is the mean and σ^2 is the variance.

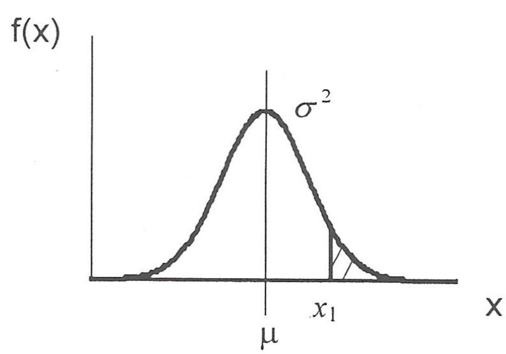

Figure 7: Normal Distribution and Point x_1

To estimate the area under the curve for the value more than x_1, the use of standard normal table N(0,1) or any statistical software is possible. In the standard normal table (also known as z-distribution), the variable z is introduced which is defined as

$$z = \frac{x_1 - \mu}{\sigma} \tag{1}$$

Therefore

$$P\{x \geq x_1\} = P\ \{ z = \frac{x_1 - \mu}{\sigma} \} \equiv \Phi\ (\frac{x_1 - \mu}{\sigma}) \tag{2}$$

Where Φ (.) is the cumulative distribution function of the standard normal distribution (μ=0, σ =1). For the standard normal N(0,1), Equation (2) is reduced to

$$P\{x \geq x_1\} \equiv \quad \Phi\ (x_1) \tag{3}$$

The idea of control charts is to estimate whether a point is from a stable process, which follows a standard normal distribution. Therefore, the probability of point x_1 not from normal distribution could be calculated by subtracting it from 1 as

$$\text{Prob}_{\ x1not1} = 1 - \Phi\ (x_1)$$
$$\text{Prob}_{\ (1point\ Est)} = 1 - \Phi\ (x_1) \tag{4}$$

To make sure that the estimates is robust for points that appear on the left or the right sides of the normal curve the absolute value of x is used to calculate Φ. Therefore the final equation is

$$\text{Prob}_{\ (1point\ Est)} = 1 - \Phi\ (|x_1|) \tag{5}$$

2.1.2 Two Points Estimate (2PE)

Similarly for 2PE the probability of the points from normal distribution is estimated. The sequence of these two points is also considered in the probability estimation.

Suppose points x_1 and x_2 appear as shown in Figure 8.

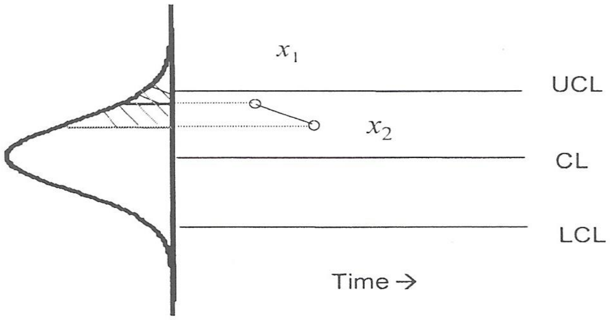

Figure 8: Two Points Estimates

The probability of these points could be estimated as

$$P\{x \geq x_1\} = P\{ z = \frac{x_1 - \mu}{\sigma} \} \equiv \Phi (\frac{x_1 - \mu}{\sigma}) \qquad (6)$$

For point x_1, the probability is estimated with $\Phi(x_1)$
For point x_2, the probability is estimated with $\Phi(x_2)$
The probability of x_1 and x_2 appearing together could also be estimated as

$$P(x_1) \cap P(x_2) = \Phi(x_1).\ \Phi(x_2) \qquad (7)$$

The probability estimates by considering the sequence of points x_1 and x_2 are explained as follows. There are two possible sequences in which points x_1 and x_2 could possibly appear.

Sequence I: Assume first point = x_1, second point = x_2,

Using Equation (7), the probability of the points appearing in sequence I could be estimated as

$$\text{Prob (Not Sequence I)} = \overline{P(x_1) \cap P(x_2)} = 1 - \Phi(x_1).\ \Phi(x_2) \qquad (8)$$

Sequence II: Assume first point = x_2, second point = x_1,
Equation (7) can also be used to determine the probability of the points appearing in this sequence as

$$\text{Prob (Not Sequence II)} = \overline{P(x_2) \cap P(x_1)} = 1 - \Phi(x_2).\ \Phi(x_1) \qquad (9)$$

The probability of the above mentioned sequences can be estimated as
Probability (sequence I or II) = 1- Prob (Not Sequence I and Not Sequence II) (10)

Upon substituting Equation (8) and (9) into Equation (10) , the following equation is obtained

$$\text{Probability (sequence I or II)} = 1 - [1 - \Phi(x_1).\ \Phi(x_2)]^2 \qquad (11)$$

Probability (sequence I or II) not from Normal Distribution is estimated as

$$\text{Prob}_{\text{x1and x2not2}} = [1 - \Phi(x_1).\ \Phi(x_2)]^2 \qquad (12)$$

Therefore, the final estimates of probability of these two points would be as

$$\text{Prob}_{\text{(2points Est)}} = [1 - \Phi(x_1).\ \Phi(x_2)]^2 \qquad (13)$$

The final estimate will also take into the consideration points that might appear on the left side of the normal distribution in which

$$\text{Prob}_{(2\text{points Est})} = [\,1 - \Phi(|x_1|).\ \Phi(|x_2|)]^2 \quad (14)$$

It can be seen that the right hand side of Equation (14), has a power of 2. The power 2, indeed, indicates the number of possible combination for two points. Another way to calculate the possible combinations is by calculating the factorial of the points, i.e. 2! = 2x1 =2.

2.1.3 Three Points Estimate (3PE)

For 3PE the whole algorithm mentioned above for 2PE is repeated that will result in the following probability estimates.

$$\text{Prob}_{(3\text{points Est})} = [1 - \Phi(|x_1|).\ \Phi(|x_2|).\ \Phi(|x_3|)]^6 \quad (15)$$

In Equation (15), x_1 is the first point, x_2 is the second point and x_3 is third point.

2.1.4 Four Points Estimate (4PE)

Similarly, for 4PE, the probability is estimated as

$$\text{Prob}_{(4\text{points Est})} = [1 - \Phi(|x_1|).\ \Phi(|x_2|).\ \Phi(|x_3|).\Phi(|x_4|)]^{24} \quad (16)$$

In Equation (16), x_1 is the first point, x_2 is the second point, x_3 is third point and x_4 is the last point.

2.2 Graded Severity Ranking (GSR)

As mentioned earlier, the probability calculated is in the form of, 0.9999 etc. To simplify the number so that it is easier for engineers to understand and interpret, a Graded Severity Ranking (GSR) is introduced. These numbers are assigned to the probability calculated based on the One Point to Four Point Estimates algorithms. The maximum of the probability estimates from 1PE until the 4PE is used as the basis for the calculation of the GSR.

The GSR is just a representation of the severity of the estimated probability of the points appearing on a control chart. It is proposed that this GSR is calculated based on the inverse of the cumulative distribution function (Φ^{-1}) of the standard normal distribution as in Equation (17).

$$\text{GSR} = \Phi^{-1}[\max(\text{Prob}_{(1\text{point Est})}, \text{Prob}_{(2\text{points Est})}, \text{Prob}_{(3\text{point Est})}, \text{Prob}_{(4\text{point Est})})] \quad (17)$$

where, $\text{Prob}_{(1\text{point Est})}$, $\text{Prob}_{(2\text{points Est})}$, $\text{Prob}_{(3\text{point Est})}$, $\text{Prob}_{(4\text{point Est})}$ are estimates of 1PE. 2PE, 3PE and 4PE respectively.

2.3 Illustrative Sample Calculation from the Proposed Methodology

Figure 9 shows a control chart with four points, P1, P2, P3, and P4.

For point P1, being the latest point in the control chart, the probability that this point is not from normal distribution is calculated from Equation (5).

$$\begin{aligned}\text{Prob}_{(1\text{point Est})} &= 1 - \Phi(|x_1|)\\ &= 1 - \Phi(|2.5|)\\ &= 1 - 0.00621\\ &= 0.99379032\end{aligned}$$

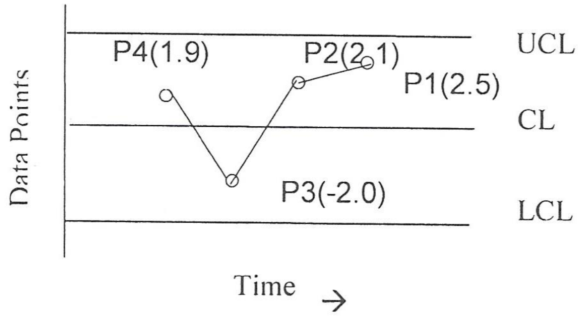

Figure 9: Control Chart for Sample Calculation

Now P1 is evaluated together with P2. The 2PE methodology is used and the probability that these points are not from normal distribution is calculated from Equation (14).

$$\text{Prob}_{(2\text{points Est})} = [\,1 - \Phi(|x_1|).\ \Phi(|x_2|)]^2$$
$$= [\,1 - \Phi(|2.5|).\ \Phi(|2.1|)]^2$$
$$= 0.999778148$$

Similarly, P1 is evaluated together with P2 and P3. Using the 3PE methodology the probability that these points are not from normal distribution is calculated from Equation (15).

$$\text{Prob}_{(3\text{points Est})} = [1 - \Phi(|x_1|).\ \Phi(|x_2|).\ \Phi(|x_3|)]^6$$
$$= [1 - \Phi(|2.5|).\ \Phi(|2.1|).\ \Phi(|-2|)]^6$$
$$= 0.999984858$$

Lastly, P1 is evaluated together with P2, P3 and P4 used and the probability that these points are not from normal distribution is calculated from Equation (16).

$$\text{Prob}_{(4\text{points Est})} = [1 - \Phi(|x_1|).\ \Phi(|x_2|).\ \Phi(|x_3|).\Phi(|x_4|)]^{24}$$
$$= [1 - \Phi(|2.5|).\ \Phi(|2.1|).\ \Phi(|-2|).\Phi(|1.9|)]^{24}$$
$$= 0.999998261$$

The summary of the estimating methodology is summarized in Table 1.

Table 1: Summary of the results of calculation based on all of the methodology

One Point Estimate	Two Points Estimates	Three Points Estimates	Four Points Estimates
0.99379032	0.999778148	0.999984858	0.999998261

Using Equation (17), the GSR is estimated as:

$$\text{GSR} = \Phi^{-1}[\max(\text{Prob}_{(1\text{point Est})}, \text{Prob}_{(2\text{points Est})}, \text{Prob}_{(3\text{point Est})}, \text{Prob}_{(4\text{point Est})})]$$
$$= \Phi^{-1}[\max(0.99379032, 0.999778148, 0.999984858; 0.9999982610]$$
$$= \Phi^{-1}[0.9999982610]$$
$$= 4.6$$

Now 4.6 is the calculated GSR assigned to P1. Similarly, when new points appear in the control chart, the same estimation could be done.

3.0 RESULTS ON SAMPLE APPLICATION OF THE PROPOSED METHODOLOGY

Figure 10 shows 100 points that are generated based on N(0.5,1). The line chart is data points or quality characteristic plotted in a control chart while the bar columns are plotted based on the GSR estimates. Referring to Figure 10, area A is examined. Area A is further exploded into Figure 11. Figure 10 shows the data points of the quality characteristic in line chart while the bottom chart is the GSR in bar chart. The control chart is further divided into area B and C.

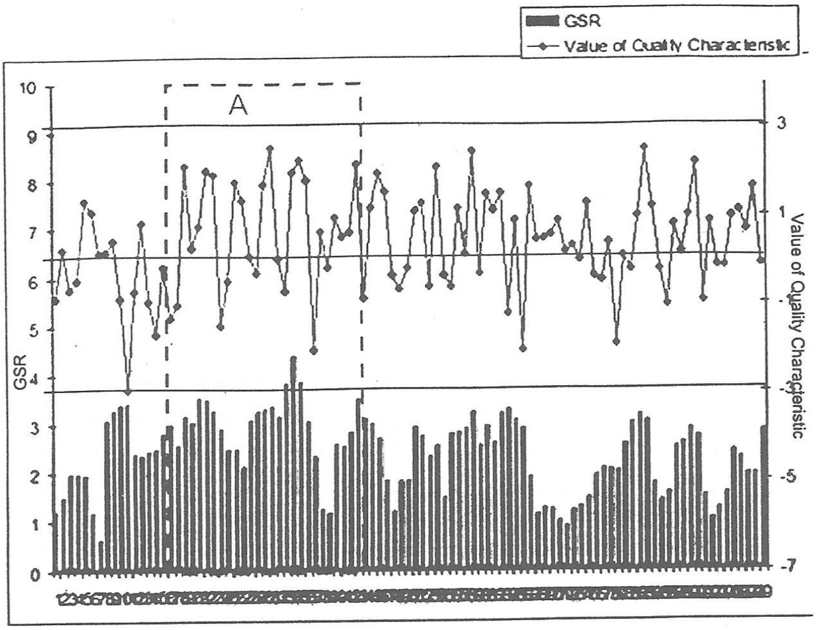

Figure 10: 100 points from N (0.5,1)

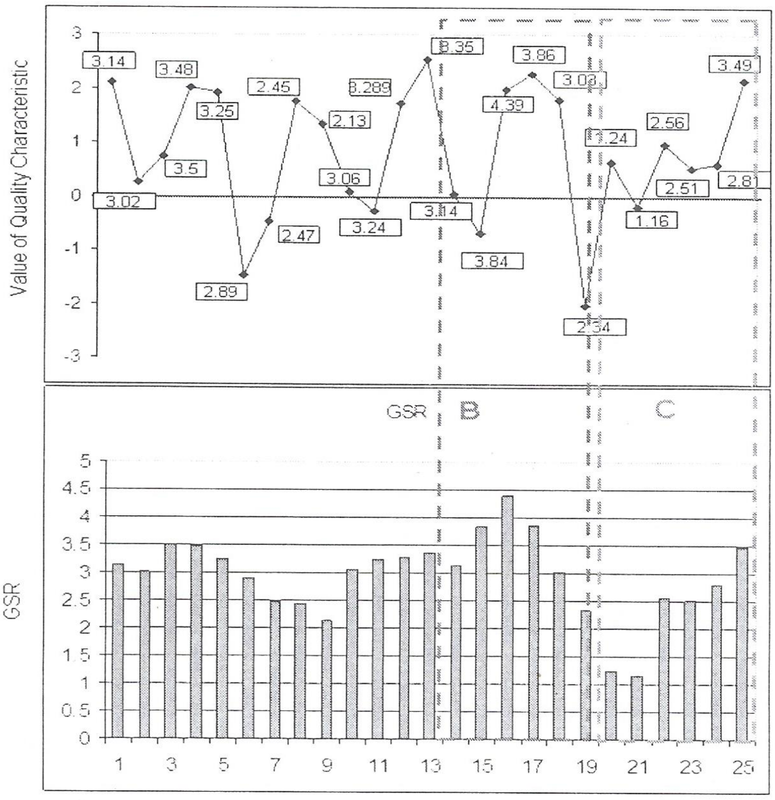

Figure 11: Exploded View of Area A

In area B, it is observed that these points are at the high point of the chart. This is intentionally done because the data is from N(0.5,1). Therefore the estimated GSRs are hovering around at 3.5. This is also very consistent with points that appear on the chart. While in area C, the points are approximately centered on zero. Here the GSR estimations are less than 2.5.

4.0 APPLICATION OF PROPOSED METHODOLOGY ON SIMULATION OF N(0.5,1)

Using Minitab, 5,000 data are generated using the Standard normal distribution N(0.5,1). Table 2 is the summary of the number of alarms generated from 8 WECO rules. From table 2, it is observed that 37 alarms are generated for run-rule 1. Run rule 1 is defined as "One point more than 3.00 standard deviations from center line". Now, this set of data is assigned with a GSR based on the proposed estimate methodology. The advantage of assigning these ranks is that it is possible now to know the severity of each alarm relative to one another. As mentioned earlier, 37 points violate the run rule 1, but with the proposed methodology, it is possible to rank which out of the 37 alarms are most or least severe.

Table 2: Summary of alarms generated by WECO Run Rule for N (0, 1)

Run Rule	Number of Alarms
1	37
2	207
3	1
4	20
5	48
6	129
7	0
8	4

The details of the alarms are from point numbered as:
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:

```
104, 152, 432, 528, 794, 958, 1139, 1170, 1563, 1572,
1818, 1853, 1942, 1970, 1982, 2046, 2240, 2301, 2460,
2580, 3040, 3106, 3163, 3214, 3573, 3649, 3669, 3692,
3726, 3825, 4073, 4140, 4244, 4260, 4496, 4911
```

Table 3 summarizes the 37 points that are violating the run rule 1. From those 37 points the GSR could be estimated and from the GSR it is now possible to rank the severity of alarms relative to each other.

Table 3: Summary of ranking between the 37 alarms

Point Numbered	Value of Quality Characteristic	Ranking
3825	3.19617	4.6
4496	3.90893	4.6
104	3.0351	4.6
1942	4.04779	4.5
3163	3.97739	4.5
1572	4.00071	4.4
3106	-3.51601	4.4
3669	3.10515	4.3
2046	3.56179	4.2
958	3.07377	4.2
2460	3.04356	4.2
1982	3.01343	4.1
1853	3.23424	4.0
1818	3.80447	4.0
4911	3.16561	4.0
1970	3.07295	3.8

Continue:

Point Numbered	Value of Quality Characteristic	Ranking
4140	3.13637	3.8
152	3.09478	3.8
794	3.31527	3.7
1139	3.39487	3.7
432	3.29639	3.7
3726	3.31688	3.7
4260	3.03887	3.6
3573	3.15583	3.6
3040	3.01547	3.6
3692	3.0585	3.6
2301	3.069	3.5
1170	3.06646	3.5
1563	3.07867	3.5
2580	3.22412	3.5
3649	3.28692	3.4
4073	3.19873	3.4
528	3.19766	3.4
4244	3.38534	3.4
3214	3.3352	3.4
2240	3.11659	3.2

4.1 Discussion on Results

By applying the proposed methodology the GSR could be estimated for each point. Table 3 shows the summary of all the points that has violated the Run Rule 1. A total of 37 alarms were flagged. It is possible to rank all of the 37 alarms in terms of severity relative to one another. Higher GSR will indicate more severe alarms.

Note that the GSR number for point numbered 3825 with the quality characteristic value of 3.19617 is at 4.6 while point numbered 1942 with quality characteristic value of 4.04779 is at 4.5. The observation is that the GSR value of point 3825 is higher than point 1942 although the value of the quality characteristic for the former is lower than the latter. This scenario is possible because the algorithm used to estimate the GSR is also considering sequence and value of that point with some of the adjacent points. As for the point numbered 3106 with quality characteristic value of -3.51601, the estimated GSR is at 4.4. Note that the value of this quality characteristic is negative. It can be observed that the estimation methodology using GSR is independent of the sides (upper side or lower side with CL= 0) how the point appear in the control chart. For this set of data, it is also observed that there are very little points that is on the negative side.

This is because the simulated data is using N(0.5,1). The data has a shift on mean 0.5 to the upper side.

In practice, after familiarizing with the GSR the engineer will be accustomed with the GSR numbers. His approach on different GSR will be different based on his experience he develops while using the GSR numbers. For example, he will hypothetically inspect five numbers of possible causes for GSR of 3 compared to seven different possible causes for a GSR of 4. It might also be reverse when higher GSR will result in the engineer having to check less possible causes. A significantly high GSR will result in very obvious causes namely measurement system error or systematic data collection error. The engineers' response and pattern of thinking will be based on the GSR and will develop as he uses and gets accustomed to the processes. He must also relearn the interpretation of the GSR on new processes.

5.0 CONCLUSION

In this paper, one of the limitations of the traditional control charts is discussed. The limitation is that because traditional control chart uses zones as a decision point, whenever a point is in a particular zone, it is considered to violate a rule. However, the degree of severity is not being considered. This paper proposed a methodology to estimate a GSR to all points in the control chart. A case study is used by performing a simulation on N(0.5,1). From the N(0.5,1) there are 37 alarms that violate the Run Rule 1 (from the WECO Rule). With the proposed methodology it is now able to rank the severity of the 37 alarms between one another. This methodology can be used to complement the widely used WECO Rules.

REFERENCES

1. Shewhart, W.A., (1931), "*Economic Control of Quality of Manufactured Product*". New York, D.Van Nostrand Company. Inc
2. Montgomery, D.C., (2001), "*Introduction to Statistical Quality Control*" 4th Edition, New York, John Wiley & Sons. Inc.
3. Margavio, T.M, Conerly, M.D, Woodall, W. H and Drake L.G, (1995), "*Alarm Rates for Quality Control Charts*". Elsevier, Statistic and Probability Letters, 24, pp. 219-224.
4. Bai, D.S and Lee, K.T, (1998), "*An Economic Design of Variable Sampling Intervals Xbar control charts*". International Journal Production Economics 54, pp. 57-64.
5. Reynolds, M.R., Jr, and Lu, C.W, (1997), "*Control Chart for Monitoring Processes With Auto correlated Data, Non Linear*", Theory and Method & Applications Vol 30, pp. 4059-4067, Proc 2nd World Congress of Non- Linear Analysis, Elsevier Science Ltd.

6. Chan, L.Y, Lai, C.D, Xie. N and Goh, T.N, (2003), "*A Two Stage Decision Procedure for Monitoring Processes with Low Fraction Non Conforming*". European Journal of Operational Research, 150, pp. 420- 436.

BIBLIOGRAPHY

1. Monk, J.D., (1969), "*Introduction to Set Theory*". New York McGraw-Hill Book Company.
2. Minitab R14., (1972 – 2003), Minitab Inc.
3. SAS Intitute, (1989), "*JMP Help File Version 4.0.2.* 2000". SAS Institute Inc
4. Shewhart, W.A., (1939), "*Statistical Method from the Viewpoint of Quality Control*" (Dover edition) reprinted from Graduate School of the Department Agriculture, New York, Dover Publication Inc.