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ASSESSMENT OF LATTICE BOLTZMANN SIMULATION SCHEME IN PREDICTING TWO-PHASE (SOLID-FLUID) FLOW

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ABSTRACT

In this paper, the coupled lattice Boltzmann simulation scheme with second Newton's law is proposed to predict the behavior of a solid particle in lid-driven cavity flow. The lattice Boltzmann scheme alone is first performed to characterize the fluid flow at Reynolds number of 100 and 400. Comparisons with the benchmark results demonstrate the applicability of the method to reproduce complex fluid structure in the system. The same density of buoyant particle is then inserted in the cavity, and its transient orbit at Reynolds number of 130 is plotted. Although the initial trajectories are found slightly deviate from the experimental results due to initial transient error, the general pattern is considered to be in close agreement with those published in the literatures.

Keywords : *Lattice Boltzmann, second Newton's law, solid particle, lid-driven cavity flow.*

1.0 INTRODUCTION

The phenomenon of multiphase flow can be seen not only in daily live situations but also in almost all engineering applications. The importance in understanding this problem results in many technical papers appear in recent years discussing its impacts on engineering. Among the researches pertinent to this problem, very few researchers devoted their study on the interaction between solid particles and fluid flow. Interestingly, this type of multiphase fluid flow plays an important role in the seeds drying technology, separation of grains, productions of milk powder, fluidized beds, coal combustion and many others.

The authors believe that the main reason of lack of understanding on the fluid-solid interaction phenomenon is the complicated nature of the problem. The size of solid particles can be as big as grains seed or very tiny such as dust pollutant. Till present day, most researchers rely on computational rather than experimental approach to study the behaviour of these particles in fluid flow. To the best of authors' knowledge, only Tsorng *et. al.* [1] reported details experimental results on the behavior of solid particles in lid-driven cavity flow from micro to macro size of particle. According to their paper, high accuracy of laser equipments together with high-speed digital image capture and data interpretation system are required to obtain reliable experimental data. Such these high cost experimental devices will not be affordable if not supported by research fund. As an alternative approach, most researchers considered fully computational scheme in their investigations. Kosinski *et. al.* [2][3][4] provides extensive numerical results on the

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subject. From the behaviour of one particle in a lid-driven cavity flow to thousands of particles in expansion horizontal pipe have been studied in their research works sheds new hope in understanding this problem. Kosinski *et al* applied the combination of continuum Navier-Stokes equations to predict fluid flow and second Newton's law for solid particle. As their model predicts excellent results when compared to the experimental results, however, the complicated nature of Navier-Stokes equation demands high computational time in resolving fluid part. High computational grid is required as the size of particle becomes smaller in order to correctly capture the position of particle in the system. In contrast, the mathematical foundation of lattice Boltzmann method (LBM) [5] makes it a suitable tool for fluid-solid interaction prediction.

LBM foundation adopts the kinetic theory of gases which considers the evolution of fluid based on the behaviour at molecular level [6][7]. Accordingly, LBM resolves the macroscale of fluid flow indirectly by solving the evolution equation of particle distribution function and models the propagation and collision of particle distribution which are believed to be the fundamental behaviours at molecular level [8]. From this similarity between the mechanisms of LBM and the behaviour of solid particle, it is considered that the LBM is the best choice to couple with the second Newton's law for prediction of fluid-solid interaction.

2.0 MATHEMATICAL FORMULATION

The lattice Boltzmann Method (LBM) involves the evolution equation of single particle distribution function *f* and can be written as [9][10][11]

$$\frac{\partial f_i}{\partial t} + \mathbf{c}_i \frac{\partial f_i}{\partial \mathbf{x}} = -\frac{f_i - f_i^{eq}}{\tau} \tag{1}$$

where f and f_i^{eq} are the density and equilibrium density distribution functions. c_i is the lattice velocity and i is the lattice direction, Δt is the time interval, and τ is the relaxation times of the density distribution function. In LBM, the magnitude of c_i is set up so that in each time step Δt, the distribution function propagates in a distance of lattice nodes spacing Δx. This will ensure that the distribution function arrives exactly at the lattice nodes after Δt and collides simultaneously.

The macroscopic variables such as the density ρ and velocity of fluid $\boldsymbol{u}$ can be computed in terms of the particle distribution functions as

$$\rho = \sum_i f_i$$
$$\mathbf{u} = \sum_i \mathbf{c}_i f_i \Big/ \sum_i f_i \tag{2}$$

To simulate fluid flow, one often uses the D2Q9 model [12], a nine lattice velocities assigned on a two-dimensional square lattice. These velocities include eight moving velocities along the links connecting the lattice nodes of the square lattice and a zero velocity for the rest particle. The rest particle is defined by the distribution functions f_0, the particles moving in the orthogonal direction by the function f_i (i = 1,2,3,4) and the particles moving in the diagonal directions by the function f_i (i = 5,6,7,8).

The equilibrium distribution functions f_i^{eq} is given as

$$f_i^{eq} = \rho w_i \left[1 + 3c_i.u + 4.5(c_i.u\ )^2 - 1.5u^2\ \right] \quad (3)$$

where ω is the weight function and depends on the direction of the lattice velocity.

Through the multiscaling expansion, the mass and momentum equations can be derived for the D2Q9 model of the evolution equation of the density distribution function. Details derivation can be found in [13]. The time relaxation, τ in lattice Boltzmann formulation can be related to the fluid viscosity in the macroscopic world as follow

$$\tau = 3\upsilon + 0.5 \quad (4)$$

In present investigation, we only consider one particle in a lid driven cavity and assume the presence of solid particle gives no effect to the fluid flow. The equation of motion for solid particle is written as

$$m_p \frac{d\mathbf{v}_p}{dt} = \mathbf{f}_p \quad (5)$$

where m_p, $\mathbf{v}_p$ and $\mathbf{f}_p$ are the mass of particle, its velocity and drag force acting on particle due to surrounding fluid. According to Kosinski *et.al.*, the drag force can be written as follow

$$\mathbf{f}_p = C_D A_p \rho \frac{|\mathbf{u} - \mathbf{v}_p|(\mathbf{u} - \mathbf{v}_p)}{2} \quad (6)$$

where A_p is the projected area of solid particle and $C_D$is the drag coefficient which is given as

$$C_D = \frac{24}{Re_p} \quad (7)$$

The particle's Reynolds number in the above equation is calculated as follow

$$Re_p = \frac{d_p|\mathbf{u} - \mathbf{v}_p|}{\upsilon} \quad (8)$$

where d_p is the diameter of solid particle.

In summary, the evolution of the scheme consists of three steps. Once the initial values of $\mathbf{u}$, $\mathbf{v}_p$, m_p and initial position of solid particle (x_p, y_p) are specified, then the system evolves in the following steps.

i) The drag force acting on solid particles is calculated from eqs. (6-8).

ii) Since the pre-calculated value of $\mathbf{v}_p^n$ is known at previous time step, the new value of particle's velocity $\mathbf{v}_p^{n+1}$ can be calculated from eq. (5) as follow

$$m_p \frac{\mathbf{v}_p^{n+1} - \mathbf{v}_p^n}{\Delta t} = \mathbf{f}_p$$

iii) Finally, the new position of solid particle can be determined as follow

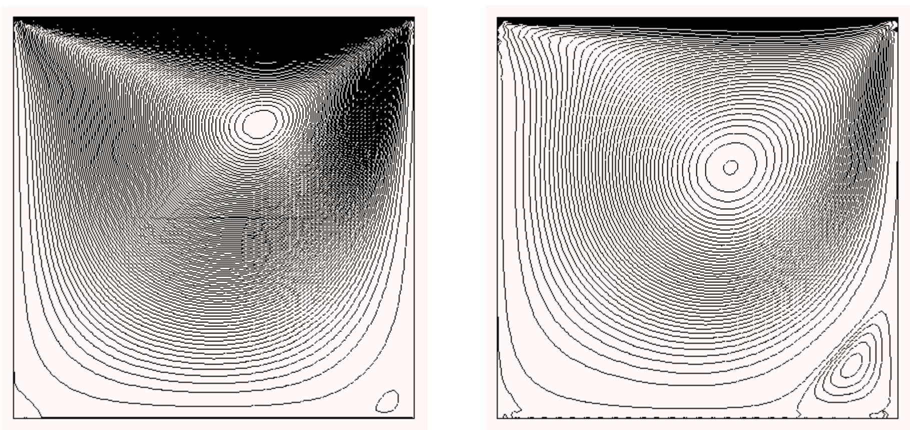
$$\mathbf{x}_p^{n+1} = \mathbf{v}_p^{n+1}\Delta t + \mathbf{x}_p^n$$

3.0 NUMERICAL SIMULATIONS

The simulation of lid-driven cavity flow start with the absence of solid particle in the system. The simulations were carried out at two values of fluid's Reynolds number which are 100 and 400 and defined as

$$Re = \frac{UL}{\upsilon} \tag{9}$$

where U, L and υ are the velocity of the top lid, width of the cavity and fluid's viscosity respectively. Figure 1 shows the computed streamline for these two cases. For *Re* = 100, the center of vortex is located at about one-third of the cavity depth from the top. As *Re* increase, the primary vortex moves towards the center of cavity and increasing circular. In addition to the primary, a pair of counter rotating eddies develop at the lower corners of the cavity and getting bigger in size when *Re* increases. In order to validate the computed results, we compared the velocity profile at the mid-width and mid-height of the cavity with the benchmark results provided by Ghia *et. al*. [14]. Figure 2 shows the comparisons of velocity profiles and excellent agreements is obtained, providing some further confidence in the approach.



(a) *Re* = 100 (b) *Re* = 400

Figure 1: Streamline plots for two values of Reynolds numbers.

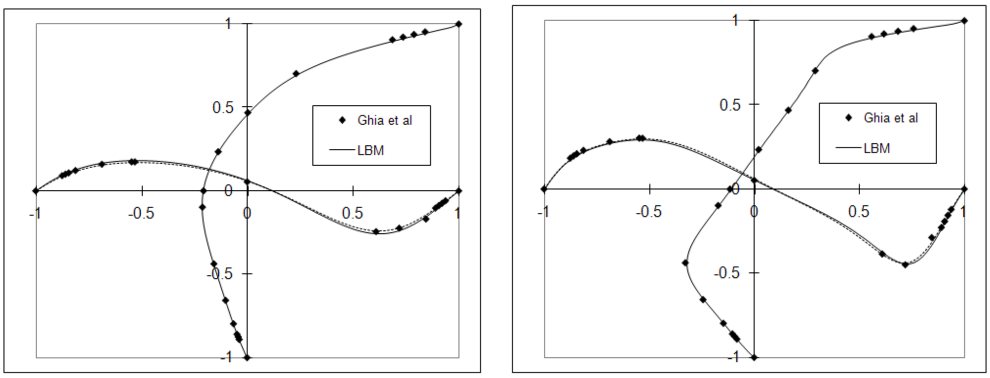


Figure 2: Comparisons of velocity profiles between LBM and benchmark results for Re = 100(left) and Re = 400 (right).

In the next prediction, the solid particle with 3-unit diameter is positioned in a 100-unit of square cavity near the top lid. The top lid of the cavity is constantly moved to the right so that the dimensionless magnitude of Reynolds number will be 130. The position of particle at every time step is recorded and plotted in Figure 3. The result obtained by experimental approach published by Tsorng *et.al*. [1] is brought for the sake of result's comparison. As can be seen from the figures, except for a short interval of time around the starting point, the predicted orbits are quite similar in character.

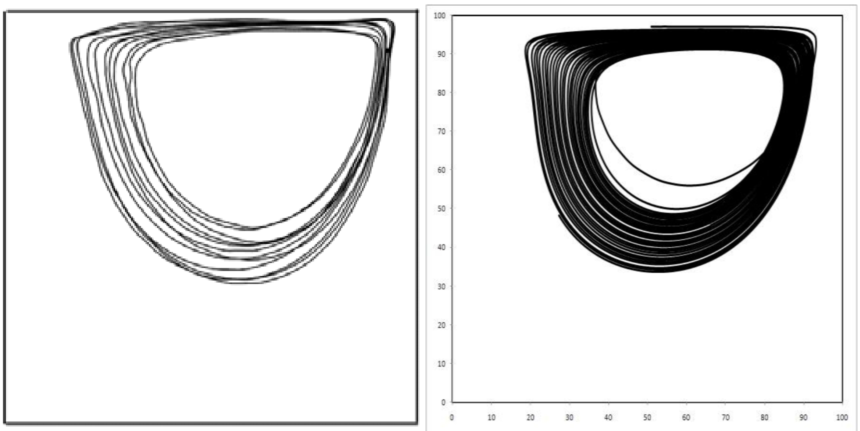


Figure 3: Orbits of particle plotted from experimental investigation of Tsorng *et.al* [1] (left) and current method (right).

4.0 CONCLUSIONS

In this paper, the behavior of solid particle in lid-driven cavity flow was predicted using the lattice Boltzmann method and second Newton's law. Validation of computer code was first carried out to characterize the flow field without the presence of solid particle. Simulation at Reynolds numbers of 100 and 400 correctly predicted the flow structure and corner eddies in the system. Simulation with the presence of solid particles was conducted at Reynolds number of 130. Good agreements were obtained with the experimental results when the plotted particle's orbit at transient condition was compared. It is concluded that the presented results support the idea that the lattice Boltzmann simulation scheme contains sufficient qualities to predict fluid-solid interaction.

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