# Computerized Analysis of Superheated Rankine Cycle

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## ABSTRACT

*Thermodynamic property equations obtained from the literature have been used to simulate the performance of a superheated Rankine cycle, with water as the working fluid. The equations were embedded in a computer program, where the inputs are condenser pressure, boiler pressure, turbine inlet temperature, and isentropic efficiency of the turbine. The program outputs are the heat supply at the boiler, network output, thermal efficiency and specific steam consumption. A case study demonstrates the effects of the condenser pressure, boiler pressure, and turbine inlet temperature on the cycle performance. The program outputs replicate the well-known facts, where thermal efficiency increases with a decrease in condenser pressure and it increases with an increase in the turbine inlet temperature.*

**Keywords:** *Rankine cycle, thermal efficiency, specific steam consumption*

## 1.0 INTRODUCTION

Steam power plant is used widely for generating the electrical energy consumed in the domestic, commercial and industrial sectors. The simplest thermodynamic cycle to model the actual, more complex cycle is known as the *Rankine* cycle. The *Rankine* cycle is made up of four basic components: a boiler feed water pump, a boiler, a turbine, and a condenser. Water is predominantly used as the working fluid in the actual plants. The fuels used in the large-scale plants include coal and low-grade oils.

The performance of a *Rankine* cycle can be analyzed manually, making use of tabulated thermodynamic properties which are included in most of the Thermodynamics textbooks. However, rigorous parametric analysis of the cycle is too tedious when done manually, and computerized analysis is therefore highly desirable. Computerized simulation requires property equations for steam, which are available in various degree of complexities [1–5].

In this work, accurate thermodynamic property equations for water have been obtained from the literature, and when an equation is not available, least-square regression of tabulated steam data is made to produce the required correlation. The independent variables (properties) needed to simulate the *Rankine* cycle are condenser pressure, boiler pressure, turbine inlet temperature, and isentropic efficiency of the turbine. The outputs from the simulation are heat supply at the boiler, network output of the cycle, thermal efficiency, and specific steam consumption.

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Computerized simulation also enables a rigorous study to be made on the effects of the different combinations of the operating variables on the performance of the cycle, quickly and accurately. In addition, simulation work is inexpensive and safe, compared to laboratory or full scale experimental work which involves very high pressures and temperatures.

## 2.0 RANKINE CYCLE

Figure 1 shows the schematic diagram of a *Rankine* cycle which is made up of a boiler, a turbine, a condenser and a boiler feed water pump. The corresponding cycle on a T-s diagram is shown in Figure 2. The analysis follows the standard practices and convention that can be found in thermodynamics textbooks [6–8].

Superheated steam enters the turbine at state 1, and exits the turbine at state 2 as a mixture of saturated liquid and vapor. Steam expansion in the turbine produces useful work output where the rotating shaft drives an electrical generator to produce electricity. Heat is rejected from the steam to the surroundings at the condenser, converting saturated vapor to saturated liquid. A boiler feed water pump then draws low-pressure saturated liquid from the condenser at state 3, and elevates the pressure to the boiler pressure a state 4, which is the highest pressure in the cycle. At the boiler, heat is added at constant pressure, producing high-pressure superheated steam at state 1, thus completing the cycle.

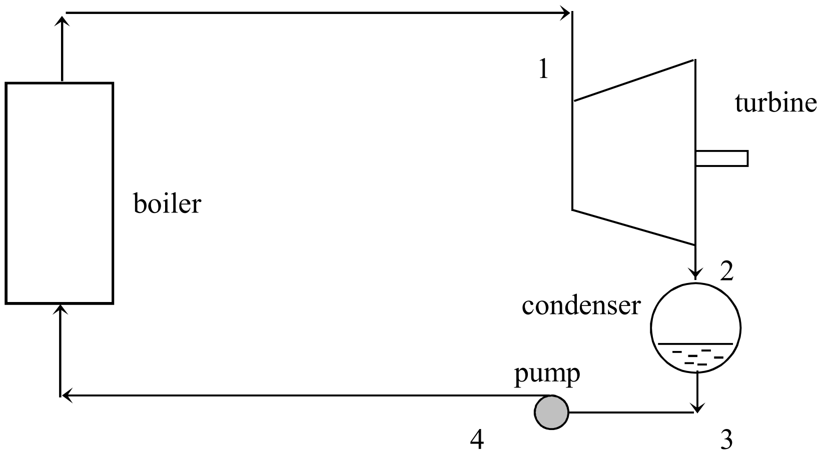


**Figure 1**: Schematic of a *Rankine* cycle

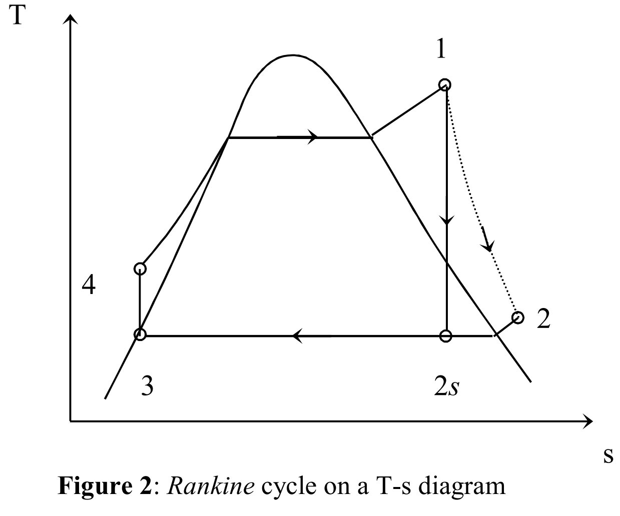


**Figure 2**: *Rankine* cycle on a T-s diagram

The analysis of the cycle is usually made with the aid of steam tables. In this article, the property equations were used, in lieu of the tabulated steam data to analyze the superheated *Rankine* cycle. This approach helps to expedite rigorous analysis, by simultaneously varying one or more property combination in one simulation. Computerized analysis can also be seen as a digital form of the experimental work.

### 2.1 Property Equations and Cycle Analysis

At the turbine inlet, the steam enthalpy is obtained from Irvine and Liley [9]. In the superheat region, the property equations are complex because they must represent the changing nature of the intermolecular forces from the saturated vapor line to the perfect gas region [9]. In this region, the pressure and temperature are designated as independent variables. The equations can be obtained by using *Taylor's* series expansion from the saturation line, and using a function which is multiplied by a factor $\exp[(ts - t)/m]$, where $ts$ is the saturation temperature at the given pressure, $t$ is the superheated steam temperature and $m$ is a constant. The factor vanishes in the region far from the saturation region, and the remaining terms will then account for the perfect gas behavior [9]. The steam enthalpy equation is thus:

$$h_1 = \mathrm{a0} + \mathrm{a1}*t_1 + \mathrm{a2}*t_1^2 - \mathrm{a3}*\exp[(t_{sb} - t_1)/45.0] \quad (1)$$

where

$\mathrm{a0} = \mathrm{b11} + \mathrm{b12}*p_b + \mathrm{b13}*p_b^2$
$\mathrm{a1} = \mathrm{b21} + \mathrm{b22}*p_b + \mathrm{b23}*p_b^2$
$\mathrm{a2} = \mathrm{b31} + \mathrm{b32}*p_b + \mathrm{b33}*p_b^2$
$\mathrm{a3} = \mathrm{b41} + \mathrm{b42}*t_{sb} + \mathrm{b43}*t_{sb}^2 + \mathrm{b44}*t_{sb}^3 + \mathrm{b45}*t_{sb}^4$
b11 = 2041.21
b12 = – 40.40021
b13 = – 0.48095
b21 = 1.610693
b22 = 5.472051e-2
b23 = 7.517537e-4
b31 = 3.383117e-4
b32 = – 1.975736e-5
b33 = – 2.87409e-7
b41 = 1707.82
b42 = – 16.99419
b43 = 6.2746295e-2
b44 = – 1.0284259e-4
b45 = 6.4561298e-8
$p_b$ : boiler pressure (MPa)
$t_{sb}$ : saturation temperature at $p_b$ (K)
$t_1$ : steam temperature at turbine inlet (K)

The steam entropy at turbine inlet is also obtained from Irvine and Liley [9]:

$$s_1 = \mathrm{a0} + \mathrm{a1}*t_1 + \mathrm{a2}*t_1^2 + \mathrm{a3}*t_1^3 + \mathrm{a4}*t_1^4 + \mathrm{b1}*\ln(10*p_b + \mathrm{b2}) - (\mathrm{c0} + \mathrm{c1}*t_{sb} + \mathrm{c2}*t_{sb}^2 + \mathrm{c3}*t_{sb}^3 + \mathrm{c4}*t_{sb}^4)*\exp[(t_{sb} - t_1)/85] \quad (2)$$

where

a0 = 4.6162961
a1 = 1.039008e-2
a2 = – 9.873085e-6

a3 = 5.43411e-9
a4 = – 1.170465e-12
b1 = – 4.650306e-1
b2 = 0.001
c0 = 1.777804
c1 = – 1.802468e-2
c2 = 6.854459e-5
c3 = – 1.184424e-7
c4 = 8.142201e-11

The entropy at the outlet of an isentropic turbine is simply, $s_{2s} = s_1$. The entropy was then compared with the saturated vapor entropy at the condenser pressure, obtained from Affandi *et al*. [10] to determine the phase at 2*s*, with an average error of 0.04%. This is a modified form of the *Wagner and Pruss* model [11].

$$s_g = \exp[1.47735 + 0.53242*\log(1/t_r)^{0.35} - 0.01923/t_r^2 + 0.02974/t_r^4 - 0.00802/t_r^5] \quad (3)$$

where
$t_r$ : reduced condenser temperature, $t_r = t_{sc} / 647.096$
$t_{sc}$ : saturation temperature at condenser pressure (K)

The saturation temperature, $t_{sc}$ at condenser pressure, $p_c$ is obtained from Irvine and Liley [9]. This equation is a modified form of the *Clausius-Clapeyron* equation:

$$t_{sc} = a + b/(\ln(p_c) + c) \quad (4)$$

where
$p_c$ : condenser pressure (MPa)

When $p_c$ is less than 12.33, the coefficients are:

a = 42.6776
b = – 3892.7
c = – 9.48654

and for $p_c$ greater than or equal to 12.33, then:

a = – 387.592
b = – 12587.5
c = –15.2578

The saturation pressure at boiler pressure, $t_{sb}$ is also obtained from Equation (4), by replacing $t_{sc}$ with $t_{sb}$, and $p_c$ replaced with $p_b$.

When $s_{2s}$ is greater than $s_g$ then the steam is in the superheated vapor region, and the enthalpy is calculated from the following equation obtained from multilinear regression of tabulated steam data from [6].

$$h_{2s} = a0 + a1*p_r + a2*p_r^2 + a3*s + a4*s^2 + a5*p_r*s \quad (5)$$

where
$p_r$ : reduced condenser pressure, $p_r = p_c/22064$.
$p_c$ : condenser pressure (kPa).

The coefficients in Equation (5) depend on the reduced condenser pressure. When the reduced pressure is $0.0004532 \leq p_r < 0.004532$ (i.e., $10 \leq p_c < 100$ kPa) and $s_{g,pc} < s < 8.5$ kJ/kgK, then:

a0 = – 274.663
a1 = 104993
a2 = – 0.218222e8
a3 = 330.702
a4 = 1.08283
a5 = 12865.5

whereas for reduced pressure $0.004532 \leq p_r \leq 0.0136$ (i.e. $100 \leq p_c \leq 300$) and $s_{g,pc} < s < 7.5$ kJ/kgK:

a0 = – 1702.15
a1 = 25205.9
a2 = – 0.121937e7
a3 = 742.137
a4 = – 23.3768
a5 = 2755.53

with a maximum error of 0.11% in both cases. When the state 2*s* is in the mixture region, then the quality of the mixture is:

$$x_{2s} = (s_{2s} - s_{f,pc}) \,/\, s_{fg,pc} \tag{6}$$

where

$$s_{f,pc} = s_g - h_{fg}/t_{sc} \tag{7}$$

and $s_g$ is given by Equation (3). The enthalpy of the vaporization, $h_{fg}$ at a reduced condenser pressure, $p_r$ is obtained from the linear regression of the tabulated steam data from Cengel and Boles [6] as follows:

$$h_{fg} = 2433.27 - 0.107559\text{e}6{*}p_r + 0.341333\text{e}8{*}p_r^2 - 0.655322\text{e}10{*}p_r^3 + 0.630003\text{e}12{*}p_r^4 - 0.234288\text{e}14{*}p_r^5 \tag{8}$$

when $p_r$ is less than 0.009065:

$$h_{fg} = 2291.09 - 0.946559\text{e}4{*}p_r - 0.234608\text{e}6{*}p_r^2 + 0.273467\text{e}8{*}p_r^3 - 0.819105\text{e}9{*}p_r^4 + 0.832313\text{e}10{*}p_r^5 \tag{9}$$

when $p_r$ is between 0.009065 and 0.033992:

$$h_{fg} = 2216.85 - 0.582147\text{e}4{*}p_r + 0.377665\text{e}5{*}p_r^2 - 0.188225\text{e}6{*}p_r^3 + 0.494692\text{e}6{*}p_r^4 - 0.521687\text{e}6{*}p_r^5 \tag{10}$$

when $p_r$ is between 0.033992 and 0.271936:

$$h_{fg} = 1686.42 + 0.152183\text{e}4{*}p_r - 0.122456\text{e}5{*}p_r^2 + 0.241571\text{e}5{*}p_r^3 - 0.222049\text{e}5{*}p_r^4 + 0.752471\text{e}4{*}p_r^5 \tag{11}$$

when $p_r$ is between 0.271936 and 0.815809, with a maximum error of 0.11% in all cases, the specific enthalpy of the mixture is calculated from:

$$h_{2s} = h_f + x_{2s}\, h_{fg} \tag{12}$$

where the enthalpy of the saturated liquid at condenser pressure is given by:

$$h_f = h_g - h_{fg} \tag{13}$$

The enthalpy of the saturated vapor, $h_g$ at reduced condenser temperature, $t_r$ is obtained from [10] with a maximum error of 0.26% and an average error of 0.05%:

$$h_g = \exp\,[[64.87678 + 11.76476{*}\ln(1/t_r)^{0.35} - 11.94431/t_r^2 + 6.29015/t_r^3 - 0.99893/t_r^4\,]^{0.5}] \tag{14}$$

When the turbine is not isentropic, the actual enthalpy at the turbine outlet is calculated from:

$$h_2 = h_1 - \eta_T\,(h_1 - h_{2s}) \tag{15}$$

where $\eta_T$ is the turbine isentropic efficiency. The boiler feed water pump work input is obtained from:

$$w_p = 0.001(p_b - p_c) \tag{16}$$

where the boiler and condenser pressures are in kPa. The enthalpy at the boiler feed water pump outlet is obtained from:

$$h_3 = h_f + w_p \tag{17}$$

The heat input, $q_{in}$ and the network output, $w_{net}$ are calculated as follows:

$$q_{in} = h_1 - h_3 \tag{18}$$

$$w_{net} = (h_1 - h_2) - w_p \tag{19}$$

The thermal efficiency is given by:

$$\eta_{th} = w_{net} / q_{in} \tag{20}$$

and the specific steam consumption is:

$$ssc = 3600/w_{net} \tag{21}$$

The preceding equations have been embedded in a *Fortran 77* program for simulating the performance of a superheated *Rankine* cycle. The program enables analysis to be made quickly to study the effects of the condenser pressure ($p_1 = p_c$), boiler pressure ($p_2 = p_b$), turbine inlet temperature($t_1$), and turbine isentropic efficiency ($\eta_t$), on the cycle performance. The limitation of the program is that the condenser pressure must be between 10 and 300 kPa, as set by Equation (5). This limitation can be removed by adding new correlations for the condenser pressures beyond 300 kPa.

## 3.0 RESULTS AND DISCUSSION

### 3.1 Validation of Computer Program

Table 1 shows the results obtained from Cengel and Boles [6] and those predicted by the computer program developed in this study where $p_1$ = 10 kPa, and $\eta_T$ = 100%. It shows that the outputs from the computer program are in excellent agreement with the published results.

**Table 1**: Comparison between published and computer results (present work)

| Cengel and Boles [1] | Present work | Percentage difference |
|---|---|---|
| | $p_2$ = 3000 kPa, $t_1$ = 350°C | |
| $q_{in}$ = 2921.3 | $q_{in}$ = 2930.1 | 0.3% |
| $w_{net}$ = 977 | $w_{net}$ = 982.1 | 0.5% |
| $\eta_{th}$ = 33.4% | $\eta_{th}$ = 33.5% | 0.3% |
| | $p_2$ = 3000 kPa, $t_1$ = 600°C | |
| $q_{in}$ = 3488.0 | $q_{in}$ = 3485.2 | –0.1% |
| $w_{net}$ = 1299.5 | $w_{net}$ = 1297.5 | –0.2% |
| $\eta_{th}$ = 37.3% | $\eta_{th}$ = 37.2% | –0.3% |
| | $p_2$ = 15000 kPa, $t_1$ = 600°C | |
| $q_{in}$ = 3376.2 | $q_{in}$ = 3370.6 | –0.2% |
| $w_{net}$ = 1452.7 | $w_{net}$ = 1431.7 | –1.4% |
| $\eta_{th}$ = 43.0% | $\eta_{th}$ = 42.5% | –1.2% |

### 3.2 Case Study

A case study has been made to analyze the effects of the condenser pressure, boiler pressure, and turbine inlet temperature on the performance of a simple *Rankine* cycle. Figure 3 shows the effects of the condenser pressure and boiler pressure on thermal efficiency, when the turbine inlet temperature was fixed at 300°C, and turbine efficiency is constant at 85%. It shows that, for a given boiler pressure, a low condenser pressure is desirable, because it gives a high thermal efficiency. At a given condenser pressure, there is also an optimum boiler pressure where the thermal efficiency is the highest. In practice, there is a limit to the lowest condenser pressure, dictated by the saturation temperature at condenser pressure which should be about 10 K higher than the heat sink medium, such as sea water to which heat is being rejected [6]. This is to ensure effective heat transfer and practical size of the condenser.

Figure 4 gives the specific steam consumption (*ssc*) variation with boiler pressure, at condenser pressures of 10, 20 and 30 kPa. It shows that a low condenser pressure will produce low *ssc*, which indicates a smaller boiler is needed to raise steam for a net power output of 1 kW. It is also seen that for a given condenser pressure, there is an optimum boiler pressure which will give the minimum *ssc*.

The positive effect of low condenser pressure on thermal efficiency and *ssc* is negated by the reduced steam quality (increased wetness) at the turbine outlet as shown in Figure 5. Reduced steam quality due to lowering of the condenser pressure is not desirable, because it increases the chances of the turbine blade to get damaged [6].

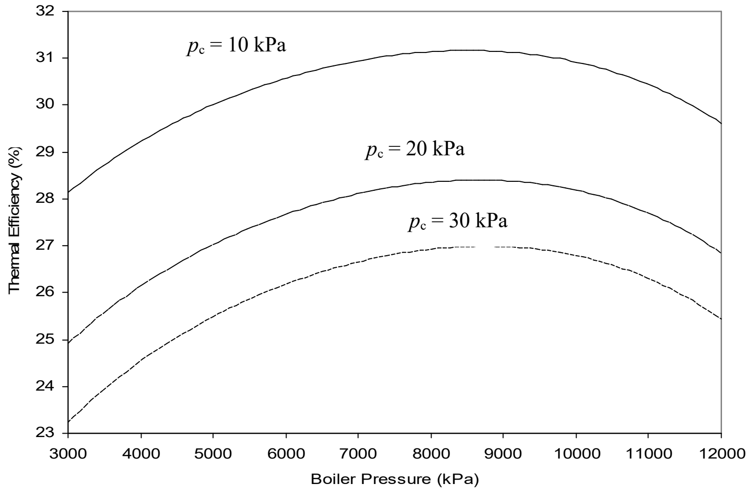


**Figure 3**: *Thermal efficiency* vs *boiler pressure* ($t_1$ = 300$^o$C, $\eta_T$ =85%)

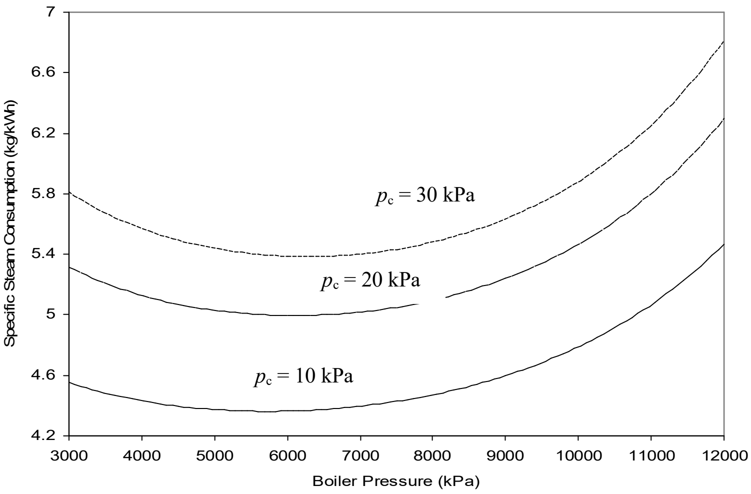


**Figure 4***: Specific steam consumption* vs *boiler pressure* ($t_1$ = 300$^o$C, $\eta_T$ =85%)

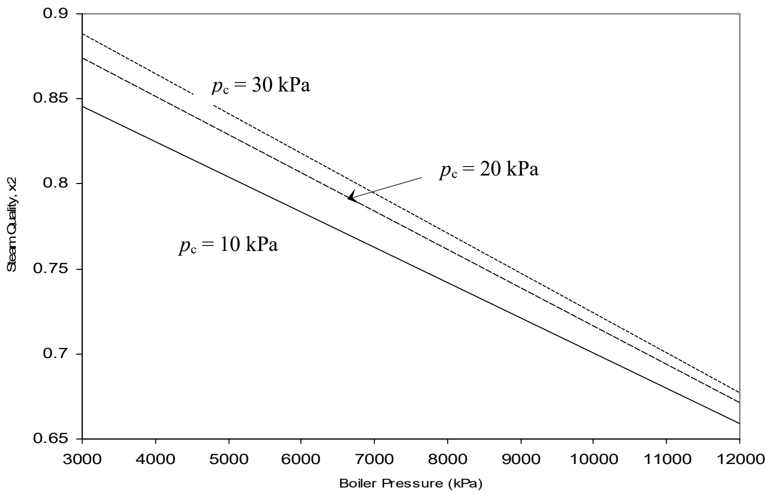


**Figure 5**: *Steam quality at turbine outlet* vs *boiler pressure* ($t_1$ = 300$^o$C, $\eta_T$ =85%)

Figure 6 shows the effect of the turbine inlet temperature ($t_1$) on the thermal efficiency, at various boiler pressures. It shows that when $t_1$ is below about 350ºC, the thermal efficiency increases with the boiler pressure, reaches a maximum value and then decreases with a further increase in the boiler pressure. However, when $t_1$ is above 350ºC, the thermal efficiency always increases with the boiler pressure. In general, at a given boiler pressure, an increase in $t_1$ will result in an increase in the thermal efficiency. The increment is small for the boiler pressures less than 8 MPa, but becoming more substantial at a boiler pressure of above 10 MPa.

Figure 7 shows the specific steam consumption (*ssc*) variation with the boiler pressure, at the turbine inlet temperatures of 310, 330, 350 and 390ºC. It can be seen that at any given boiler pressure, an increase in $t_1$ effectively lowers the *ssc*. The decrement is small for boiler pressure in the 6 to 10 MPa range, but becomes more substantial as the boiler pressure increases beyond 8 MPa. In general, for a fixed turbine inlet temperature, the *ssc* increases with an increase in boiler pressure when $t_1$ is below 390ºC. The increment is substantial as $t_1$ becomes much smaller than 390ºC. The *ssc* is almost constant at $t_1$ = 390ºC.

An increase in $t_1$ increases the thermal efficiency, and lowers the *ssc*, both of which are positive effects on the plant performance. The additional benefit to increasing $t_1$ is that the steam quality at the turbine outlet increases at any given boiler pressure as shown in Figure 8. The other trend is that for a fixed $t_1$, an increase in the boiler pressure reduces the steam quality. An increase in the steam quality (reduced wetness) is good for turbine operation, because there is a lower tendency for the turbine blades to get damage. In practice, there is an upper limit to turbine inlet steam temperature, dictated by the metallurgical strength of the turbine blades, where the current highest temperature is about 620ºC [6].

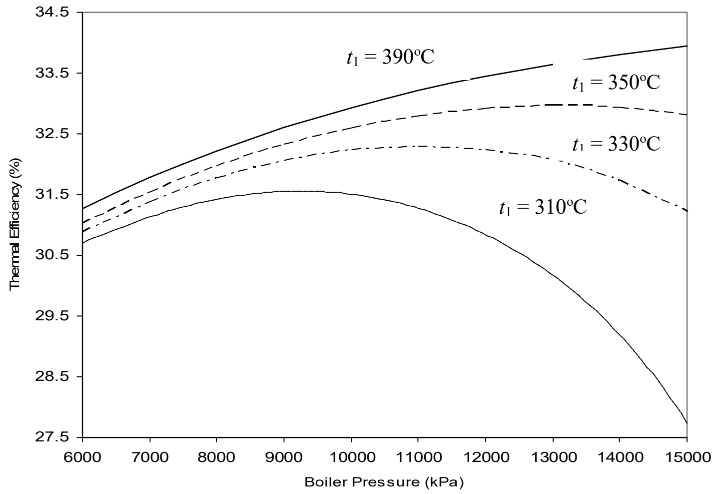


**Figure 6**: *Thermal efficiency* vs *boiler pressure* ($t_1$ = 310, 330, 350, 390ºC; $\eta_T$ = 85%; $p_c$ =10 kPa)

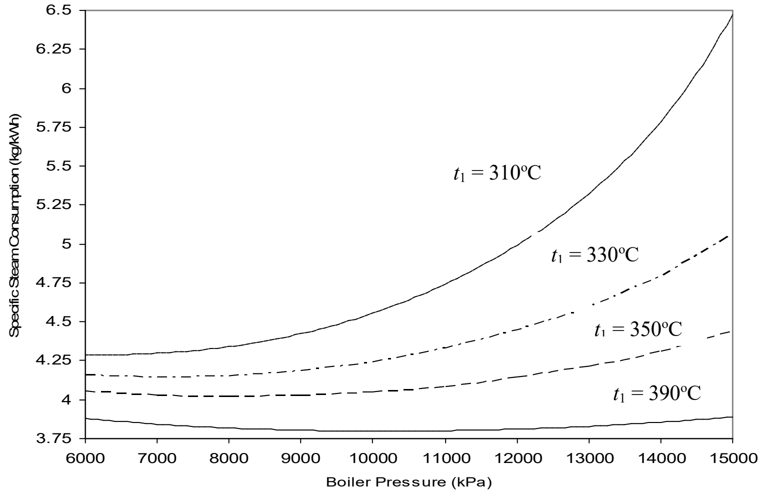


**Figure 7**: *Specific steam consumption* vs *boiler pressure* ($t_1$ = 310, 330, 350, 390°C; $\eta_T$ = 85%; $p_c$ = 10 kPa)

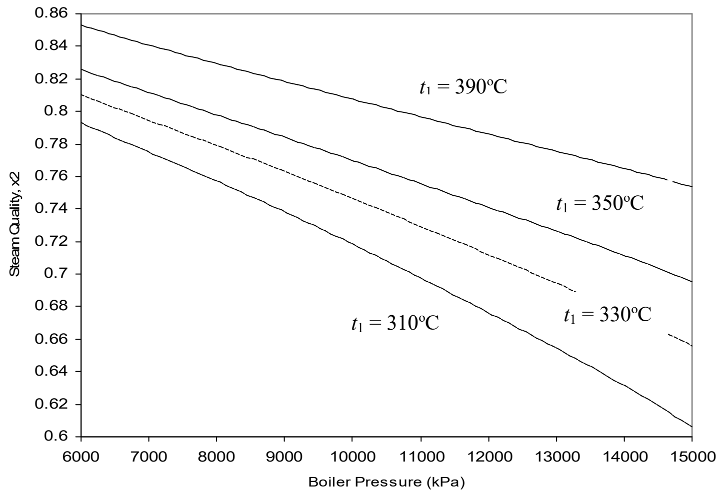


**Figure 8**: *Steam quality at the turbine outlet* vs *boiler pressure* ($t_1$ = 310, 330, 350, 390°C; $\eta_T$ = 85%; $p_c$ = 10 kPa)

## 4.0 CONCLUSION

A comprehensive set of accurate thermodynamic property equations have been compiled, and used to develop a computer program for simulation of a basic steam power plant based on the superheated *Rankine* cycle. The computer program enables the effects of the different combination of operating variables (condenser pressure, boiler pressure, turbine inlet temperature, and turbine isentropic efficiency) on plant performance to be made. The key performance indicators computed by the program include thermal efficiency, specific steam consumption and steam quality at turbine outlet. The case study gives findings that are consistent with the widely known general trends for the superheated *Rankine* cycle, discussed in standard thermodynamics textbooks. The trends are, the thermal efficiency always increases with reduced condenser pressure, and it also increases with an increase in the turbine inlet temperature. When the boiler pressure is increased, the thermal efficiency increases, reaches a maximum value, and then decreases, when the turbine inlet temperature is below a certain threshold value. Above this threshold value, the thermal efficiency always increases with an increase in the boiler pressure.

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